Exercise 59

Find equations of both lines that are tangent to the curve $y = x^3 - 3x^2 + 3x - 3$ and are parallel to the line $3x - y = 15$.

Solution

Writing the given equation of the line as

$$y = 3x - 15,$$

we see that it has a slope of 3. The aim is to take the derivative of the given function and find where it's equal to 3.

$$\begin{aligned}
y' &= \frac{d}{dx}(x^3 - 3x^2 + 3x - 3) \\
&= \frac{d}{dx}(x^3) - \frac{d}{dx}(3x^2) + \frac{d}{dx}(3x) - \frac{d}{dx}(3) \\
&= \frac{d}{dx}(x^3) - 3\frac{d}{dx}(x^2) + 3\frac{d}{dx}(x) - \frac{d}{dx}(3) \\
&= (3x^2) - 3(2x) + 3(1) - (0) \\
&= 3x^2 - 6x + 3
\end{aligned}$$

Set this equal to 3 and solve for x.

$$\begin{aligned}
3x^2 - 6x + 3 &= 3 \\
3x^2 - 6x &= 0 \\
3x(x - 2) &= 0 \\
x &= \{0, 2\}
\end{aligned}$$

Plug these values of x into the given function to get the corresponding y-values on the curve.

$$\begin{aligned}
y(0) &= (0)^3 - 3(0)^2 + 3(0) - 3 = -3 \quad \Rightarrow \quad (0, -3) \\
y(2) &= (2)^3 - 3(2)^2 + 3(2) - 3 = -1 \quad \Rightarrow \quad (2, -1)
\end{aligned}$$

Finally, determine the equation of the line with slope 3 that goes through the point $(0, -3)$.

$$y + 3 = 3(x - 0)$$

And determine the equation of the line with slope 3 that goes through the point $(2, -1)$.

$$y + 1 = 3(x - 2)$$